A COMPARATIVE STUDY OF MAKE-TO-ORDER, MAKE-TO-STOCK, AND VENDOR MANAGED INVENTORY REPLENISHMENT SYSTEMS

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ABSTRACT

This paper investigates the benefit of placing a two echelon serial supply chain under centralized control through the use of a Vendor Managed Inventory (VMI) program. Under VMI, a vendor uses customer inventory information to make all replenishment decisions. Probability models are developed to compare the performance of a VMI system to traditional Make-to-Order and Make-to-Stock replenishment systems. Numerical analysis is utilized to compare the performance of these replenishment strategies under various production rates, order quantities, levels of demand variance, and shipping lead times.

Keywords: Vendor Managed Inventory, Supply Chain Management

INTRODUCTION

In an uncoordinated supply chain a manufacturer may follow either a Make-to-Order or a Make-to-Stock strategy to satisfy customer orders. Vendor Managed Inventory (VMI), an alternative replenishment strategy to MTO and MTS, coordinates the supply chain by placing it under centralized control. Under VMI, the manufacturer observes the customer’s inventory levels and is responsible for making all the customer’s replenishment decisions. As a result, the manufacturer is better able to match supply with demand by improving the timing of production and shipments.

This paper compares the performance of VMI, MTO, and MTS in a two-stage serial supply chain where a single manufacturer replenishes a single customer. The manufacturer is modeled as having a fixed production rate, $p$, and ships only complete orders. The manufacturer incurs no setup and produces batches of size $Q$. When a shipment is sent to the customer, the transportation lead time, $L$, is constant. When end consumer demand exceeds available inventory, the excess demand is backordered and is satisfied at the next replenishment.

Probability models based on renewal theory are used to establish the performance of the three replenishment models. A process is said to ‘renew’ when it is in a pre-specified state that has the important characteristic that the probabilities of future events are the same each time this point is reached. For the MTO and MTS environments, the renewal point is the point when the customer’s inventory position reaches $s_1$, the customer reorder point, and when the manufacturer has zero units (MTO) or $Q$ units (MTS) of inventory. For VMI, a renewal occurs when the manufacturer has no inventory and the customer’s inventory is $s_1$, the manufacturer’s production signal.
Since the manufacturer’s shipments are constrained to be precisely Q units, it is possible the customer will place an order before the manufacturer’s previous production run is completed. When this occurs the shipment is delayed until the manufacturer has Q units available. Because shipments may be delayed, the system does not necessarily reach a renewal point at each replenishment. Thus, it is necessary to calculate the performance measures - the expected shortage per replenishment and the manufacturer’s and customer’s expected average inventory levels - for each non-renewal replenishment. Henceforth, let $Y$ represent the number of replenishments which have occurred since the last renewal.

Let $f(x_t)$ denote the normal probability density function of demand over time $t$. Here, the relevant time periods are the lead-time, $L$, the production time, $P$, and the sum of production and lead-time, $L + P$. The probability models assume demand between events is stationary, stochastic, and follows a normal distribution. Define $d$ as the single period mean and $\sigma$ as the single period standard deviation of demand. Let $D_t$ and $\sigma_t$ be defined as the mean demand and standard deviation of demand over the time period $t$.

**MAKE-TO-STOCK AND MAKE-TO-ORDER**

The probability models for the MTO and MTS strategies are nearly identical and their only difference lies in the timing of order receipts. Under MTO, once an order is placed it takes $L+P$ periods to receive the order if the manufacturer is idle. Under MTS, an order is received after $L$ periods, assuming the manufacturer has Q units available. The calculations below use the general variable $x_t$ to signify demand over time interval $t$. To calculate the performance measures for a Make-to-Order system substitute $t = L+P$ while for Make-to-Stock substitute $t = L$.

When utilizing a Make-to-Stock strategy, the manufacturer begins producing Q units once its inventory level reaches zero and then holds the inventory once production is complete. When the customer’s inventory reaches $s_1$, an order is placed which the manufacturer fulfills with its finished goods inventory. In the Make-to-Order model, an order is placed when the customer’s inventory reaches $s_1$ and the manufacturer fulfills the order by scheduling a production run since it carries no inventory.

The first performance measure determines the number of backorders that are expected for each replenishment. Let $ESPR_y$ represent the expected shortage for the $y^{th}$ production run after a renewal has occurred. Beginning at the renewal point, for $Y = 1$ the expected shortage is:

$$ESPR_1 = \int_{s_1}^{\infty} \left( x_t - s_1 \right) f(x_t) \, dx_t$$

Recall that the process may not renew after each replenishment. During the first order after a renewal, this is the case if demand during the production lead time is greater than Q. During the second order, the process does not renew if the cumulative demand during the two orders is greater than $2Q$. Let $x_y$ represent the demand during production of the $y^{th}$ production run after a renewal has occurred. By summing $ESPR_y$ across all values of $Y$, the total ESPR for each
renewal is determined. ESPR\(_2\) and ESPR\(_3\) are calculated below and for \(Y \geq 3\), the calculations follow the same pattern.

\[
ESPR_2 = \int_0^\infty \int_0^\infty \left[ x_t - \left( s_1 + Q - x_1 \right) \right] f(x_1) f(x_t) \, dx_1 \, dx_t
\]

\[ (2) \]

\[
ESPR_3 = \int_0^\infty \int_0^{2Q-x_1} \int_0^{s_1 + Q - x_1 - x_2} \left[ x_t - \left( S + Q - x_1 - x_2 \right) \right] f(x_1) f(x_2) f(x_t) \, dx_1 \, dx_2 \, dx_t
\]

\[ (3) \]

A second performance measure is the customer’s average inventory. The customer’s inventory can be divided into safety stock and cycle stock. For any value of \(Y\), it is intuitive that the average cycle stock is simply \(Q/2\). Similar to the ESPR calculations, the safety stock calculations must consider the number of orders since the last renewal. If \(SS_y\) denotes the expected safety stock for the \(y^{th}\) order after a renewal, then \(SS_y\) for \(Y = 1\) to \(3\) is shown below. Note that for \(Y > 3\), the calculations follow the same form and the total expected safety stock, \(SS\), is calculated by summing \(SS_y\) for all \(Y\).

\[
SS_1 = \int_0^\infty \left( s_1 - x_t \right) f(x_t) f(x_1) \, dx_t
\]

\[ (4) \]

\[
SS_2 = \int_0^Q \int_0^\infty \left( s_1 - x_t \right) f(x_t) f(x_1) \, dx_t \, dx_1
\]

\[ (5) \]

\[
SS_3 = \int_0^{2Q-x_1} \int_0^\infty \int_0^{s_1 + Q - x_1 - x_2} \left( s_1 - x_t \right) f(x_1) f(x_2) f(x_t) \, dx_1 \, dx_2 \, dx_t
\]

\[ (6) \]

With respect to the manufacturer’s average inventory, note that the manufacturer is either producing or is idle. During production time, the average inventory is equal to \(Q/2\) units. After production, the manufacturer immediately ships the completed order under an MTO system but holds the inventory under MTS. Thus, during the manufacturer’s idle time zero units of inventory is held under MTO and \(Q\) units are held under MTS. Since the utilization of the system, \(\lambda/p\), represents the proportion of time the manufacturer is producing, you may calculate \(I_m\) as:

\[
I_m - MTO = \frac{\lambda Q}{2p}
\]

\[ (7) \]

\[
I_m - MTS = \frac{Q(2p - \lambda)}{2p}
\]

\[ (8) \]

**VENDOR MANAGED INVENTORY**

Under VMI, the manufacturer uses customer inventory information to determine when to produce and ship. When the customer’s inventory level reaches \(s_1\) the manufacturer begins producing immediately or once it becomes idle. The manufacturer then holds the inventory until
the customer’s inventory level reaches \( s_2 \), at which point the manufacturer will ship \( Q \) units to the customer immediately or once \( Q \) units become available.

Assume the process begins at the renewal point. So long as the demand during the production run is less than \((s_1-s_2)\) the manufacturer will have \( Q \) units on hand when the customer’s inventory reaches \( s_2 \). Define the expected shortage in this situation as \( \text{ESPR}_{1a} \). If demand is greater than \((s_1-s_2)\), the manufacturer will not have \( Q \) units available when the \( s_1 \) is reached. Define \( \text{ESPR}_{1b} \) as the expected shortage when this occurs. Similar to the MTS and MTO cases, for the process to renew the demand during the first production run must be less than \( Q \). For the first two production runs the calculations for \( \text{ESPR} \) are shown below and the same logic applies to values of \( Y > 2 \). The final \( \text{ESPR} \) is calculated by summing across all \( Y \).

\[
\text{ESPR}_{1a} = \int_0^{s_2-s_1} \int_{s_2}^{\infty} \left[ x_L - (s_2) \right] f(x_L) f(x_1) \, dx_L \, dx_1
\]

(9)

\[
\text{ESPR}_{1b} = \int_{s_1-s_2}^{\infty} \int_{s_1-x_1}^{\infty} \left[ x_L - \left( s_1 - x_1 \right) \right] f(x_L) f(x_1) \, dx_L \, dx_1
\]

(10)

\[
\text{ESPR}_{2} = \int_0^Q \int_{s_1+Q-x_1}^{\infty} \left[ x_L - \left( s_1 + Q - x_1 \right) \right] f(x_L) f(x_1) \, dx_L \, dx_1
\]

(11)

The average customer inventory calculations are shown below for \( Y = 1 \) and 2. Define \( \text{SS}_{ya} \) as the case where demand during the production run is less than \((s_1-s_2)\) and \( \text{SS}_{yb} \) when demand is greater than \((s_1-s_2)\). The calculations for \( Y > 2 \) follow along the same lines.

\[
\text{SS}_{1a} = \int_0^{s_1-s_2} \int_0^{s_2} \left[ (s_2) - x_L \right] f(x_1) f(x_L) \, dx_L \, dx_2
\]

(12)

\[
\text{SS}_{1b} = \int_{s_1-s_2}^{\infty} \int_0^{\infty} \left[ (s_1 - x_1) - x_L \right] f(x_1) f(x_L) \, dx_L \, dx_1
\]

(13)

\[
\text{SS}_{2} = \int_0^Q \int_0^{s_1+Q-x_1} \left( s_1 + Q - x_1 - x_{LP} \right) f(x_1) f(x_{LP}) \, dx_1 \, dx_{LP}
\]

(14)

For VMI, the average manufacturer inventory calculation must consider three distinct periods. Besides holding \( Q/2 \) units of inventory over the production time, under VMI the manufacturer holds \( Q \) units from the time production is complete until the customer’s inventory level reaches \( s_1 \) and holds no inventory from the time a shipment is made until the customer’s inventory position reaches \( s_2 \).
\[ I_m = \frac{\lambda Q}{2p} + \left( 1 - \frac{\lambda}{p} \right) \int_0^{s_1-s_2} \left( \frac{s_1 - xp - s_2}{\lambda} \right) f(x_p) \, dx_p \]

\[ \int_0^Q \left( \frac{Q - xp}{\lambda} \right) f(x_p) \, dx_p \]

**METHOD OF EXAMINATION**

As production rate approaches the demand rate, it becomes more likely that a process will not reach the renewal point on a given production run and the number of production runs between renewals increases. This makes it necessary to calculate performance measures for large values of \( Y \). However, the probability models become complicated when \( Y \) becomes large because of the multiple integrals. Consequently, a discrete event simulation model was developed to determine the expected values for the performance measures.

The environmental factors to be explored are the order quantity, production rate, standard deviation of demand, and the lead-time. These factors were chosen because they either affect the probability of obtaining consecutive production runs or affect the responsiveness of the supply chain. Table 1 lists each factor and the levels to be used in the numerical analysis.

<table>
<thead>
<tr>
<th>Table 1 – Experimental Design</th>
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<tr>
<td><strong>Factor</strong></td>
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<tr>
<td>1. Order Quantity (Q)</td>
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<td>2. Production Rate (p)</td>
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<tr>
<td>3. Std. Dev. Of Demand (σ)</td>
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<tr>
<td>4. Lead-time (L)</td>
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The simulation calculates the performance measure values (ESPR, average manufacturer inventory, and average customer inventory) for a given level of \( s_1 \) or \( s_2 \). Of course, the resulting performance measures will differ for each replenishment strategy (VMI, MTO, MTS) which may complicate a direct comparison of the strategies. Consequently, a quadratic search algorithm is used to select the values of \( s_1 \) and \( s_2 \) which yield a specified level of ESPR. The relative performance of one strategy versus another can then be ascertained by simply comparing the inventory levels.

**PRELIMINARY RESULTS**

Figures 1 and 2 illustrate the relationship between average inventory and fill rate in two environments. Figure 1 represents a low variability environment; order and production lots are 100 units, production rate is 20 units/period, and the single period standard deviation of demand is 3. Figure 2 represents a high variability environment; order and production lots are 30 units, production rate is 11 units/period, and the single period standard deviation of demand is 8. Both environments have a demand rate of 10 units/period and a lead-time of 2 periods.
These preliminary results yield several interesting insights. First, substantial savings may be had by moving a supply chain from a Make-to-Stock to Vendor Managed Inventory configuration in low variability environments. As variability is increased, the value of placing the supply chain under centralized control decreases substantially.

**Figure 1 – Inventory and Fill Rate relationship under low variability**

Second, the results indicate that given the freedom to make replenishment decisions, it may be difficult to improve upon a Make-to-Order system. Consequently, there may be little benefit to either the manufacturer or the customer of moving to a Vendor Managed Inventory system when the supply chain currently operates under a Make-to-Order environment.

**Figure 2 – Inventory and Fill Rate relationship under high variability**

_N.B._ References available upon request from Peter A. Salzarulo